Are Trade Marks and Patents Complementary or Substitute Protections for Innovation?

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Abstract

The benefits of innovations for firms strongly depend on their ability to develop complementary appropriability means, including intellectual property (IP) rights. This paper aims at assessing the interrelated effects of two types of IP rights, namely patents and trade marks, considering them in their core function as legal protection devices. Based on a supermodularity analysis, we show that the complementary relationship between trade marks and patents is not straightforward. Depending on the levels of advertising spillovers and depreciation rate, trade marks are found to be either complementary or substitute to patents. Based on a data set encompassing the IP activity of a sample of French publicly traded firms, we find that patents and trade marks are complementary in chemical and pharmaceutical sectors, but substitute in high-tech business sectors (computer products and electrical equipment).

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 $^{^1}$ The opinions expressed in this paper are the sole responsibility of the authors and do not necessarily reflect those of the OECD or of the governments of its member countries.

1 Introduction

The benefits of product innovations for firms strongly depend on their ability to develop complementary appropriability means (Teece 1986, Levin et al. 1987, Cohen et al. 2000). Intellectual property (IP) rights are a major factor of firms' appropriability strategies. Patents, which enable the protection of new technologies, are the most obvious IP rights related to innovation and the most extensively studied in economics literature (see Griliches 1991 for a survey). But patents alone do not guarantee that the firm will benefit from innovation, which also requires the development of market-based assets to ensure the success of the commercialisation of the innovation (Rogers 1998, Jennewein 2005, Aaker 2007). Trade marks are one of these market-based assets. Several papers in recent literature mention that trade marks can be used in relation to innovative activity (Schmoch 2003, Mendonça et al. 2004, Greenhalgh and Rogers 2007). The creation of a new trade mark may enhance consumers' perception of innovative products, and may constitute a basis for advertising. Moreover, if a product is launched on the market under a certain brand name, consumers are likely to remain loyal to this pioneer brand even after competitors enter the market (Davis 2009). Trade marks and patents then constitute two distinct means of appropriating the benefits of innovation, whose effects are likely to be interrelated.

While there are a number of studies on the complementarity between technological investments and advertising or marketing investments (Hirschey 1982, Snyder and King 2007, Brekke and Straume 2008, Askenazy et al. 2010), the relationship between patents and trade marks was rarely investigated. A few papers have empirically tested the complementarity between patents and trade marks at the firm-level considering them as proxies for technological and marketing investments (von Graevenitz and Sandner 2009, Schwiebacher 2009). However those studies do not disentangle the value of the IP rights (i.e. the patent or trade mark premium) and of the value of the protected assets: the observed complementarity between IP rights mirrors the complementarity of their respective underlying investments. Somaya and Graham (2006), as far as they are concerned, have observed complementarity effects in the joint use of various IP rights (namely copyrights and trade marks in software industries), which they explain mainly by economies of scales in organisational resources deployed for IP management. None of those studies have investigated the interaction effects of IP rights in their core function as legal protection devices. Yet the effect of various protection means are likely to be interrelated. One might consider that the different types of protections tend to overlap

each other so that their marginal effect would be lower when several types of IP rights are used. Nevertheless, patent and trade mark protections are also likely to reinforce each other. Indeed, the monopoly position that is established by a patent can favour the establishment of a strong trade mark and, in return, trade marks can be used to extend the benefits of the patents. Statman and Tyebjee (1981), for example, observe that due to brand loyalty, the expiration of patents for ethical drugs has only a minor effect on their market dominance. In their words, "the patent period is used to transfer the value of the patent into the trade mark".

Through a formal approach, our paper addresses the interaction effects that occur between trade marks and patents as legal devices that enable the protection of a certain brand and a certain technology, respectively, disentangling the value of the IP rights and the value of their underlying investments. We build a basic model that encompasses the separate and combined effects on the profits of an innovating firm of using both IP rights and we analyse the conditions in which they can be considered substitute or complementary. For this we rely on the concept of supermodularity, which enables complementarities to be addressed in a discrete-choice model environment (see Milgrom and Roberts 1990, 1995). This implies modelling beforehand the impact of protecting an innovation by a trade mark. Such a theoretical approach does not exist to our knowledge in the literature.

The model consists of a duopoly in which one firm innovates (leader) and another imitates (follower). Each firm may choose to incur advertising expenditure, which enable them to build their goodwill stock, in a dynamic framework. Advertising expenditure are not entirely appropriable: competitors can benefit from the effects of advertising spillovers. Filing a patent grants the right to prevent competitors from using the patented technology. Although patents do not always make possible to perfectly exclude other firms from the market, they tend to decrease competition. Schematically, we assume that if the firm files a patent, it benefits from a monopoly power for a limited period. For trade marks, we stick to the legal definition and consider that trade marks grant the right to prevent other parties from benefiting from the reputation that has been built by the firm by creating confusion on the origin of the product. Without trade mark, advertising has the characterisits of a public good and benefits equally all firms in the market. Filing a trade mark enables the firm to appropriate part of its advertising expenditure. Besides, if the firm registers a trade mark, the reputation that is built during the monopoly period entirely benefits the monopoly firm, so that the competitor does not benefit from any spillovers of advertising that is launched during the patent period. The interaction between patents and trade marks is then characterised by

two counterbalancing effects: a substitution effect, as the trade mark has no impact on the firm's profit during the patent period, and a complementary effect, as the reputation built in the monopoly period has an impact a posteriori on the trade mark benefits after the patent has expired. The main prediction of our theoretical model is that the predominance of the complementarity or the substitution effect is not straightforward. Depending on the levels of advertising spillovers and depreciation rate, the two IP rights can be found to be either substitutes, or complements.

Using a firm-level database that encompasses the trade marking and patenting activity of a sample of publicly traded French firms, we test the complementary or substitute relationship between patents and trade marks across various sectors. We find that in chemical and pharmaceutical industries, where the depreciation rate of advertising is likely to be low and advertising spillovers high, the two IP rights tend to be complementary, whereas in high-tech business sectors (manufacture of computer, electronic and optical products and of electrical equipment), which are likely to be characterized by high advertising depreciation rates and low advertising spillovers, the two IP rights are found to be substitutes, which confirms our theoretical predictions.

The remainder of this paper is organised as follows. Section 2 lays out the theoretical framework that is used to describe the effects of trade mark and patent protections at the firm-level, from which we analytically derive some predictions on their complementary or substitute relationship. Section 3 presents our empirical strategy with which we test the model predictions and our main empirical findings. Section 4 concludes with the implications of the model.

2 Theoretical Model

2.1 General framework

The two-period game

The starting point of the model is a firm introducing an innovation, leading to the creation of a new market for a product. The innovating firm can choose to register a patent, a trade mark, or both or neither of them. IP rights related choices are considered binary: the firm can register at most one of each type of IP rights. If the innovating firm files a patent, the model has 2 distinct periods: a monopoly period under the patent protection and then a competition period, characterised

by a Cournot-type duopoly between the innovating firm (leader) and an imitating firm (follower). We assume the innovation to be instantaneously imitable, so if no patent is filed by the innovating firm, the competition starts immediately in the first period, right after the innovation is introduced.

Advertising and goodwill

Firms incur advertising expenditure, which enable them to build a goodwill, which positively affects the demand for the product. Following Nerlove and Arrow (1962), we assume that advertising expenditure are cumulative: the goodwill of the firm is supplied at each period with advertising expenditure, and depreciates at rate δ . In a two-period framework, this translates into an equation of evolution of the goodwill stock G_t from first period to second period:

$$G_2 - G_1 = a_2 - \delta G_1, \tag{1}$$

where a_2 is the amount of second period advertising expenditure and δ is the depreciation rate of advertising between the two periods. The firms only start advertising expenditure when they enter the market, so in the first period the amount of goodwill is given by $G_1 = a_1$.

Besides, we assume that advertising expenditure are not totally appropriable by firms (Friedman 1983), and are subject to spillovers. The interpretation of those spillovers is that part of the advertising launched by a firm corresponds to advertising for the product in general and not for its own brand, so that the competitor can also benefit from it. Advertising expenditure incurred by each firm therefore divide into two parts, one share benefiting all firms in the market and the other share benefiting only the firm that has incurred the advertising expenditure.

Effect of trade mark

Regarding the function of trade marks, we stick to the legal definition and consider that trade marks prevent other parties from benefiting from the firm's reputation by creating confusion on the origin of the product. We assume that in the absence of trade mark, competitors can imitate not only the functional features of the product, but also its appearance and the signs referring to the brand image (e.g. the brand name). In that case, no part of advertising expenditure can be appropriated by

the firm. Advertising expenditure and goodwill have therefore the characteristics of a public good: they benefit equally all the firms present in the market. The total amount of second period advertising expenditure benefiting the follower is then $\overline{a_2} + sa_2$ if the leader files a trade mark, where a_2 and $\overline{a_2}$ are the amount of advertising expenditure incurred by the leader and the follower in period 2 and 0 < s < 1 is the level of advertising spillovers benefiting the follower, and $\overline{a_2} + a_2$ if the leader does not file a trade mark.

A key assumption of our model is that if the leader files both a trade mark and a patent, all the advertising expenditure incurred during the patent period correspond to advertising for the brand and benefit only its own goodwill. Indeed, during the patent period the reputation of the product coincides with the reputation of the monopoly brand, so that the brand entirely captures the reputation of the product. This means that the follower will benefit from no spillover of advertising expenditure incurred during the monopoly period. To benefit from spillovers, since the respective brand images of the leader and the follower are not confusable, the follower needs first to start to advertise its product so that the customers realise that the products are identical. The advertising spillovers are then only effective in the second period when the follower enters the market. By contrast, if no trade mark is filed, the competitor can play on confusion on the appearance of the product and thus benefit from advertising spillovers of all periods, including the monopoly period, as customers will mistakenly attribute the goodwill of the leader to the product sold by the competitor.

Lastly if no patent is filed, in each period the follower benefits from a share s of the leader's advertising expenditure in case a trade mark is filed, and from the totality of the leader's advertising expenditure if no trade mark is filed.

In summary, the amount of goodwill benefiting the leader and the follower in the second period, depending on the IP strategy adopted by the leader, is the following:

		TM	No TM
PAT	Leader	$(1 - \delta) a_1 +$	$a_2 + \overline{sa_2}$
	Follower	$\overline{a_2} + sa_2$	$\overline{a_2} + (1 - \delta) a_1 + a_2$
No PAT	Leader	$a_2 + (1 - \delta) a_1 + \overline{s}$	$(\overline{a_2} + (1 - \delta)\overline{a_1})$
	Follower	$\overline{a_2} + (1 - \delta) \overline{a_1} + s (a_2 + (1 - \delta) a_1)$	$\overline{a_2} + (1 - \delta)\overline{a_1} + a_2 + (1 - \delta)a_1$

where a_2 and $\overline{a_2}$ are the advertising expenditure incurred in the second period by the leader and by the follower, respectively, a_1 and $\overline{a_1}$ are the levels of advertising

expenditure in the first period, δ is the depreciation rate of advertising over the two periods, \bar{s} is the level of advertising spillovers benefiting the leader, and s is the level of advertising spillovers benefiting the follower if the leader files a trade mark.

In the first period, the amount of goodwill benefiting the leader and the follower, depending on the IP strategy adopted is:

		ТМ	No TM	
PAT	Leader	a_1		
	Follower	-		
No PAT	Leader	$a_1 + \overline{sa_1}$		
	Follower	$\overline{a_1} + sa_1$	$\overline{a_1} + a_1$	

Inverse demand function

Following Dixit (1979), we assume that the inverse demand function facing each firm in the market is negatively related to the total amount of quantities sold. Assuming a quadratic utility function of customers, the relationship between price and quantities is linear (Dixit 1979). We then assume that advertising increases customers' willingness to pay for the product (Brady 2009), so that the goodwill stock has a positive impact on the price for a given quantity sold. The effect of goodwill stock is assumed to have decreasing marginal returns. The inverse demand function facing the leader is given by:

$$P_t = \alpha - \beta (Q_t + \overline{Q}_t) + \tau \sqrt{G_t}, \tag{2}$$

where Q_t and \overline{Q}_t are the quantities sold by the firm and its competitor in t and G_t represents the goodwill stock of the firm, with α , β and τ strictly positive parameters. The inverse demand function facing the follower is symmetrical.

Supermodularity analysis

Based on this framework, we compare the inter-temporal profits resulting from the various IP strategies to investigate the complementarity relationship between the various protection means. More specifically, we study the complementarity between trade marks and patents based on the concept of supermodularity (Milgrom and Roberts 1990 and 1995). This framework makes it possible to analyse complementarity in the context of discrete choices (in which pay-offs are not continuous); this is appropriate here since we focus on a single invention so that the firm registers at most one trade mark and one patent. The supermodularity theory states that two inputs which can be used by the firm or not are complements only if using one input while also using the other input has a higher incremental effect on performance than using one input alone (following the intuitive idea that "the whole is more than the sum of its parts").

We test the validity of the following fundamental inequality, where V is the inter-temporal profit gained from innovation and the exponents indicate the presence or not of a trade mark (TM) or a patent (PAT):

$$V^{TM,PAT} + V^{0,0} > V^{TM,0} + V^{0,PAT}. (3)$$

If this inequality is verified, the two types of IP rights are complementary, whereas if the reverse inequality is verified, they are substitutes.

2.2 Outcome of the various intellectual property strategies

Based on the above framework, we derive the outcome of the various IP strategies on the profit of the innovating firm.

2.2.1 Case of patent protection

If the leader registers a patent, its inter-temporal profit is, from (2):

$$V = \left[\alpha - \beta Q_1 + \tau \sqrt{G_1} - c\right] Q_1 - a_1 + r \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{G_2} - c\right] Q_2 - r a_2 - C_{PAT} - (1_{TM=1}) C_{TM},$$

where c is the cost of production, assumed linear, r is the discount rate between the two periods (with r > 0, decreasing with the duration of the patent), and C_{PAT} and C_{TM} correspond to the costs of filing a trade mark and a patent, respectively.

If the innovating firm files a trade mark:

Replacing G_1 and G_2 by their expressions, the intertemporal profits of the leader can be rewritten as:

$$V = \left[\alpha - \beta Q_1 + \tau \sqrt{a_1} - c\right]Q_1 - a_1 + r\left[\alpha - \beta\left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} - C_{TM}. \tag{4}$$

The inter-temporal profit of the follower is:

$$\overline{V} = r \left[\alpha - \beta \left(\overline{Q_2} + Q_2 \right) + \tau \sqrt{\overline{a_2} + s a_2} - c \right] \overline{Q_2} - r \overline{a_2}. \tag{5}$$

The model is solved through backward induction: the firms first determine their optimal levels of advertising expenditure and quantities sold in the second period considering the stock of advertising expenditure of the leader in the first period given, then the leader maximises its inter-temporal profit on the choice variables of the first period.

1st step: maximisation of the second period profits on Q_2 , $\overline{Q_2}$, a_2 and $\overline{a_2}$ considering a_1 given

The respective programs of the leader and the follower are:

$$\max_{Q_2,G_2} \left(r \Big[\alpha - \beta \Big(Q_2 + \overline{Q_2} \Big) + \tau \sqrt{(1-\delta)a_1 + a_2 + \overline{sa_2}} - c \Big] Q_2 - ra_2 \right)$$

and

$${\rm max}_{\overline{Q_2},\overline{G_2}} \Big(r \Big[\alpha - \beta \Big(\overline{Q_2} + Q_2 \Big) + \tau \sqrt{\overline{a_2} + sa_2} - c \Big] \overline{Q_2} - r \overline{a_2} \Big).$$

The system of first order conditions yields the following Nash-Cournot equilibrium:

$$\begin{cases}
Q_2^* = \frac{\alpha - \beta \overline{Q_2}^* + \tau \sqrt{(1 - \delta)} a_1 + a_2^* + \overline{s} \overline{a_2}^* - c}{2\beta} \\
\sqrt{(1 - \delta)} a_1 + a_2^* + \overline{s} \overline{a_2}^* = \frac{\tau}{2} Q_2^* \\
\overline{Q_2}^* = \frac{\alpha - \beta Q_2^* + \tau \sqrt{\overline{a_2}^* + s} a_2^* - c}{2\beta} \\
\sqrt{\overline{a_2}^* + s} a_2^* = \frac{\tau}{2} \overline{Q_2}^*
\end{cases}
\Leftrightarrow
\begin{cases}
Q_2^* = \overline{Q_2}^* = \frac{2(\alpha - c)}{6\beta - \tau^2} \\
a_2^* = \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 - \frac{1 - \delta}{1 - \overline{s}s} a_1 \\
\overline{a_2}^* = \frac{1 - s}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 + s \frac{(1 - \delta)}{(1 - \overline{s}s)} a_1
\end{cases}$$
(6)

The optimal quantities in second period are equal for the two firms. Considering $\overline{s} = s$, the optimal amount of advertising expenditure in period 2 is higher for the follower. Indeed, since the follower does not benefit from advertising expenditure

incurred in period 1, it has to catch up with the leader in order to sell at the same price (in a Cournot competition framework¹).

From first order condition on Q_2 , and the previous expression of Q_2^* and a_2^* in (6), (4) becomes:

$$V = [\alpha - \beta Q_1 + \tau \sqrt{a_1} - c]Q_1 - a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 - r\frac{1 - \overline{s}}{1 - \overline{s}s}\left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 + r\frac{1 - \delta}{1 - \overline{s}s}a_1 - C_{PAT} - C_{TM}.$$

¹In other competition frameworks, we might observe that followers do less advertising expenditure than leaders, and compensate by a significantly lower selling price of the same product, which corresponds for example to the situation of firms selling generic drugs.

2nd step: maximisation of the leader inter-temporal profit on a_1, Q_1

The system of first order conditions on Q_1 , a_1 yields:

$$\left\{ \begin{array}{l} Q_1^* = \frac{\alpha + \tau \sqrt{a_1^*} - c}{2\beta} \\ \sqrt{a_1^*} = \frac{\tau}{2\left(1 - r\frac{1 - \delta}{1 - \overline{s}s}\right)} Q_1^* \end{array} \right. \\ \Leftrightarrow \left\{ \begin{array}{l} Q_1^* = \frac{2\left(1 - r\frac{1 - \delta}{1 - \overline{s}s}\right)(\alpha - c)}{4\beta\left(1 - r\frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2} \\ a_1^* = \left(\frac{\tau(\alpha - c)}{4\beta\left(1 - r\frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2}\right)^2 \end{array} \right. .$$

The model has an interior solution if $4\beta(1-r\frac{1-\delta}{1-\overline{s}s})-\tau^2>0$ (garanteeing that Q_2^* , $\overline{Q_2}^*$, Q_1^* , $\overline{a_2}^*$ and a_1^* are positive) and $\left(\frac{4\beta(1-r\frac{1-\delta}{1-\overline{s}s})-\tau^2}{6\beta-\tau^2}\right)^2>\frac{1-\delta}{1-\overline{s}}$ (garanteeing that a_2^* is positive), i.e. if β , the negative impact of quantities on demand is large enough compared to the impact of advertising τ , and the depreciation rate of advertising δ is large enough.

The final profit of the innovating firm in case it files both a patent and a trade mark is then equal to:

$$\begin{split} V^{TM,PAT} \! = \! \beta \! \left(\frac{2 \left(1 - r \frac{1 - \delta}{1 - \overline{s}s} \right) (\alpha - c)}{4 \beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s} \right) - \tau^2} \right)^2 - \! \left(\frac{\tau (\alpha - c)}{4 \beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s} \right) - \tau^2} \right)^2 + r \beta \left(\frac{2 (\alpha - c)}{6 \beta - \tau^2} \right)^2 - r \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau (\alpha - c)}{6 \beta - \tau^2} \right)^2 \\ + r \frac{1 - \delta}{1 - \overline{s}s} \left(\frac{\tau (\alpha - c)}{4 \beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s} \right) - \tau^2} \right)^2 - C_{PAT} - C_{TM} \,. \end{split}$$

This expression simplifies into:

$$V^{TM,PAT} = (\alpha - c)^2 \left(\frac{1 - r \frac{1 - \delta}{1 - \overline{s}s}}{4\beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2} + r\beta \left(\frac{2}{6\beta - \tau^2}\right)^2 - r \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau}{6\beta - \tau^2}\right)^2 \right) - C_{PAT} - C_{TM}. \tag{7}$$

If the innovating firm does not register a trade mark

In that case the intertemporal profit expressions of the leader and the follower write:

$$V = [\alpha - \beta Q_1 + \tau \sqrt{a_1} - c]Q_1 - a_1 + r \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta Q_1 + \tau \sqrt{a_1} - c\right]Q_1 - a_1 + r \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - ra_2 - C_{PAT} + rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_1 + a_2 + \overline{sa_2}} - c\right]Q_2 - rac{1}{2} \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{(1 - \delta)a_2 + \overline{sa_2}} - c\right]Q_2$$

$$\overline{V} \! = \! r \! \left[\alpha \! - \! \beta \! \left(\overline{Q_2} \! + \! Q_2 \right) \! + \! \tau \sqrt{\overline{a_2} \! + \! (1 \! - \! \delta) a_1 \! + \! a_2} \! - \! c \right] \overline{Q_2} \! - \! r \overline{a_2}.$$

Maximising V on a_2 , Q_2 , and \overline{V} on $\overline{a_2}$, $\overline{Q_2}$ considering a_1 given yields the following Nash-equilibrium:

In the case where the leader files no trade mark, the follower relies entirely on the advertising spillovers and does not itself incur any advertising expenditure in the second period.

The intertemporal profit of the leader then writes

$$V = \left[\alpha - \beta Q_1 + \tau \sqrt{a_1} - c\right]Q_1 - a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 - r\left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 - C_{PAT}$$

Maximising V on a_1 , Q_1 yields:

$$\begin{cases} Q_1^* = \frac{\alpha + \tau \sqrt{a_1^*} - c}{2\beta} \\ \sqrt{a_1^*} = \frac{\tau}{2(1 - r(1 - \delta))} Q_1^* \end{cases} \Leftrightarrow \begin{cases} Q_1^* = \frac{2(1 - r(1 - \delta))(\alpha - c)}{4\beta(1 - r(1 - \delta)) - \tau^2} \\ a_1^* = \left(\frac{\tau(\alpha - c)}{4\beta(1 - r(1 - \delta)) - \tau^2}\right)^2 \end{cases}.$$

An solution exists on the condition that $4\beta \left(1-r\left(1-\delta\right)\right)-\tau^2>0$ (garanteeing that $Q_2^*,\ \overline{Q_2}^*,\ Q_1^*$ and a_1 are positive) and $\left(\frac{4\beta(1-r(1-\delta))-\tau^2}{6\beta-\tau^2}\right)^2>(1-\delta)$ (garanteeing that a_2^* is positive), i.e. if β , is large enough compared to τ , and δ is large enough.

The profit of the innovating firm in case it files a patent but no trade mark is then equal to:

$$\begin{split} V^{0,PAT} \! = \! \beta \Big(\frac{2(1-r(1-\delta))(\alpha-c)}{4\beta(1-r(1-\delta))-\tau^2} \Big)^2 - \Big(\frac{\tau(\alpha-c)}{4\beta(1-r(1-\delta))-\tau^2} \Big)^2 + r\beta \Big(\frac{2(\alpha-c)}{6\beta-\tau^2} \Big)^2 - r\Big(\frac{\tau(\alpha-c)}{6\beta-\tau^2} \Big)^2 \\ + r(1-\delta) \Big(\frac{\tau(\alpha-c)}{4\beta(1-r(1-\delta))-\tau^2} \Big)^2 - C_{PAT} \end{split}$$

This expression simplifies into:

$$V^{0,PAT} = (\alpha - c)^2 \left(\frac{(1 - r(1 - \delta))}{(4\beta(1 - r(1 - \delta)) - \tau^2)} + r\beta \left(\frac{2}{6\beta - \tau^2} \right)^2 - r\left(\frac{\tau}{6\beta - \tau^2} \right)^2 \right) - C_{PAT}. \tag{8}$$

2.2.2 Case without patent protection

If the innovative firm does not protect its innovation with a patent, the competition starts in the first period. Inter-temporal profits are given by:

$$V = \left[\alpha - \beta \left(Q_1 + \overline{Q_1}\right) + \tau \sqrt{a_1 + \overline{sa_1}} - c\right] Q_1 - a_1 + r \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{a_2 + (1 - \delta)a_1 + \overline{s}(\overline{a_2} + (1 - \delta)\overline{a_1})} - c\right] Q_2 - ra_2 - (1_{TM-1})C_{TM} - c$$

and

$$\overline{V} = \left[\alpha - \beta \left(Q_1 + \overline{Q_1}\right) + \tau \sqrt{\overline{a_1} + sa_1} - c\right] \overline{Q_1} - \overline{a_1} + r \left[\alpha - \beta \left(Q_2 + \overline{Q_2}\right) + \tau \sqrt{\overline{a_2} + (1 - \delta)\overline{a_1} + s(a_2 + (1 - \delta)a_1)} - c\right] \overline{Q_2} - r \overline{a_2},$$

with 0 < s < 1 in case a trade mark is filed and s = 1 in case no trade mark is filed. Maximising V on a_2 , Q_2 , and \overline{V} on $\overline{a_2}$, $\overline{Q_2}$ considering a_1 and $\overline{a_1}$ given yields the following symmetrical Nash-equilibrium:

$$\begin{cases} Q_2^* = \frac{\alpha - \beta \overline{Q_2}^* + \tau \sqrt{a_2^* + (1 - \delta) a_1 + \overline{s}(\overline{a_2}^* + (1 - \delta) \overline{a_1})} - c}{2\beta} \\ \sqrt{a_2^* + (1 - \delta) a_1 + \overline{s}(\overline{a_2}^* + (1 - \delta) \overline{a_1})} = \frac{\tau}{2} Q_2^* \\ \overline{Q_2}^* = \frac{\alpha - \beta Q_2^* + \tau \sqrt{\overline{a_2}^* + (1 - \delta) \overline{a_1} + s(a_2^* + (1 - \delta) a_1}) - c}{2\beta} \\ \sqrt{\overline{a_2}^* + (1 - \delta) \overline{a_1} + s(a_2^* + (1 - \delta) a_1)} = \frac{\tau}{2} \overline{Q_2}^* \end{cases} \\ \Leftrightarrow \begin{cases} Q_2^* = \overline{Q_2}^* = \frac{2(\alpha - c)}{6\beta - \tau^2} \\ a_2^* = \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 - (1 - \delta) a_1 \\ \overline{a_2}^* = \frac{1 - s}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 - (1 - \delta) \overline{a_1} \end{cases}$$

The intertemporal profits of the leader and the follower then write:

$$V = \left[\alpha - \beta \left(Q_1 + \overline{Q_1}\right) + \tau \sqrt{a_1 + \overline{sa_1}} - c\right] Q_1 - a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 - r\frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)a_1 + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right)^2 + r\beta \left$$

and

$$\overline{V} = \left[\alpha - \beta \left(Q_1 + \overline{Q_1}\right) + \tau \sqrt{\overline{a_1} + sa_1} - c\right] \overline{Q_1} - \overline{a_1} + r\beta \left(\frac{2(\alpha - c)}{6\beta - \tau^2}\right) - r\frac{1 - s}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta - \tau^2}\right)^2 + r(1 - \delta)\overline{a_1}.$$

Maximising V on a_1 , Q_1 and \overline{V} on $\overline{a_1}$, $\overline{Q_1}$ yields:

$$\begin{cases} Q_1^* = \frac{\alpha - \beta \overline{Q_1}^* + \tau \sqrt{a_1^* + \overline{sa_1}^*} - c}{2\beta} \\ \sqrt{a_1^* + \overline{sa_1}^*} = \frac{\tau}{2(1 - \tau(1 - \delta))} Q_1^* \\ \overline{Q_1}^* = \frac{\alpha - \beta Q_1^* + \tau \sqrt{\overline{a_1}^* + sa_1^*} - c}{2\beta} \\ \sqrt{\overline{a_1}^* + sa_1^*} = \frac{\tau}{2(1 - \tau(1 - \delta))} \overline{Q_1}^* \end{cases} \Leftrightarrow \begin{cases} Q_1^* = \overline{Q_1}^* = \frac{2(1 - \tau(1 - \delta))(\alpha - c)}{6\beta(1 - \tau(1 - \delta)) - \tau^2} \\ a_1^* = \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta(1 - \tau(1 - \delta)) - \tau^2}\right)^2 \\ \overline{a_1}^* = \frac{1 - s}{1 - \overline{s}s} \left(\frac{\tau(\alpha - c)}{6\beta(1 - \tau(1 - \delta)) - \tau^2}\right)^2 \end{cases}$$

An interior solution exists on the condition that $6\beta \left(1-r\left(1-\delta\right)\right)-\tau^2>0$ (garanteeing that Q_2^* , $\overline{Q_2}^*$, Q_1^* , $\overline{Q_1}^*$, a_1^* and $\overline{a_1}^*$ are positive) and $\left(\frac{6\beta(1-r(1-\delta))-\tau^2}{6\beta-\tau^2}\right)^2>(1-\delta)$ (garanteeing that a_2^* and $\overline{a_2}^*$ are positive), i.e. if β is large enough compared to τ , and δ is large enough.

The leader's inter-temporal profit in case it files a trade mark but no patent equals:

$$V^{TM,0} = \beta \left(\frac{2(1-r(1-\delta))(\alpha-c)}{6\beta(1-r(1-\delta))-\tau^2}\right)^2 - \frac{1-\overline{s}}{1-\overline{s}s} \left(\frac{r(\alpha-c)}{6\beta(1-r(1-\delta))-\tau^2}\right)^2 + r\beta \left(\frac{2(\alpha-c)}{6\beta-\tau^2}\right)^2 - r\frac{1-\overline{s}}{1-\overline{s}s} \left(\frac{r(\alpha-c)}{6\beta-\tau^2}\right)^2 + r\left(1-\delta\right)\frac{1-\overline{s}}{1-\overline{s}s} \left(\frac{r(\alpha-c)}{6\beta(1-r(1-\delta))-\tau^2}\right)^2 - C_{TM}$$

$$= (\alpha-c)^2 \left((1-r(1-\delta))\left(\frac{4\beta(1-r(1-\delta))-\frac{1-\overline{s}}{1-\overline{s}s}\tau^2}{(6\beta(1-r(1-\delta))-\tau^2)^2}\right) + r\beta \left(\frac{2}{6\beta-\tau^2}\right)^2 - r\frac{1-\overline{s}}{1-\overline{s}s} \left(\frac{r}{6\beta-\tau^2}\right)^2\right) - C_{TM}. \tag{9}$$

In case neither a patent nor a trade mark is filed, the expression of the intertemporal profit is the same as above, with s = 1 and no filing cost:

$$V^{0,0} = (\alpha - c)^2 \left((1 - r(1 - \delta)) \left(\frac{4\beta(1 - r(1 - \delta)) - \tau^2}{(6\beta(1 - r(1 - \delta)) - \tau^2)^2} \right) + r\beta \left(\frac{2}{6\beta - \tau^2} \right)^2 - r \left(\frac{\tau}{6\beta - \tau^2} \right)^2 \right). \tag{10}$$

2.3 Comparison of outcomes and complementarity analysis

2.3.1 Determination of the optimal IP right strategy

Based on the previous results, we seek to determine the optimal IP right strategies for innovating firms, by comparing the intertemporal profits resulting from the different IP right combinations.

From (7) and (8), we get:

$$V^{TM,PAT} - V^{0,PAT} = (\alpha - c)^{2} \left(\frac{1 - r \frac{1 - \overline{\delta}}{1 - \overline{\delta}s}}{4\beta \left(1 - r \frac{1 - \overline{\delta}}{1 - \overline{\delta}s}\right) - \tau^{2}} - r \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau}{6\beta - \tau^{2}} \right)^{2} - \left(\frac{(1 - r(1 - \delta))}{(4\beta (1 - r(1 - \delta)) - \tau^{2})} - r \left(\frac{\tau}{6\beta - \tau^{2}} \right)^{2} \right) \right) - C_{TM}$$

$$= (\alpha - c)^{2} \left(\frac{\tau^{2} r(1 - \delta) \left(\frac{\overline{s}s}{1 - \overline{s}s} \right)}{\left(4\beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^{2} \right) \left(4\beta (1 - r(1 - \delta)) - \tau^{2} \right)} + r \overline{s} \left(\frac{\tau}{6\beta - \tau^{2}} \right)^{2} \frac{1 - s}{1 - \overline{s}s} \right) - C_{TM}$$

$$= K_{1} - C_{TM}, \tag{11}$$

with $K_1 > 0$.

From (9) and (10):

$$\begin{split} V^{TM,0} - V^{0,0} = & (\alpha - c)^2 \left(\frac{(1 - r(1 - \delta)) \left(4\beta (1 - r(1 - \delta)) - \frac{1 - \overline{s}}{1 - \overline{s}s} \tau^2 \right)}{(6\beta (1 - r(1 - \delta)) - \tau^2)^2} - r \frac{1 - \overline{s}}{1 - \overline{s}s} \left(\frac{\tau}{6\beta - \tau^2} \right)^2 - \frac{(1 - r(1 - \delta)) \left(4\beta (1 - r(1 - \delta)) - \tau^2 \right)}{(6\beta (1 - r(1 - \delta)) - \tau^2)^2} + r \left(\frac{\tau}{6\beta - \tau^2} \right)^2 \right) - C_{TM} \\ = & (\alpha - c)^2 \left((1 - r(1 - \delta)) \left(\frac{\tau^2 \overline{s} \frac{1 - s}{1 - \overline{s}s}}{(6\beta (1 - r(1 - \delta)) - \tau^2)^2} \right) + r \overline{s} \left(\frac{\tau}{6\beta - \tau^2} \right)^2 \frac{1 - s}{1 - \overline{s}s} \right) - C_{TM} \\ = & K_2 - C_{TM} \,, \end{split}$$

with $K_2 > 0$. Considering sufficiently low filing costs, it is always beneficial for firms to file a trade mark.

Besides, from (7) and (9), we get

$$\begin{array}{lcl} V^{TM,PAT} - V^{TM,0} & = & (\alpha - c)^2 \left(\frac{1 - r \frac{1 - \delta}{1 - \overline{s}s}}{4\beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2} - (1 - r(1 - \delta)) \left(\frac{4\beta (1 - r(1 - \delta)) - \frac{1 - \overline{s}}{1 - \overline{s}s}\tau^2}{\left(6\beta (1 - r(1 - \delta)) - \tau^2\right)^2} \right) \right) - C_{PAT} \\ & = & (\alpha - c)^2 \left(1 - r \frac{1 - \delta}{1 - \overline{s}s}\right) \frac{\left(6\beta (1 - r(1 - \delta)) - \tau^2\right)^2 - \left(4\beta (1 - r(1 - \delta)) - \frac{1 - \overline{s}}{1 - \overline{s}s}\tau^2\right) \left(4\beta (1 - r(1 - \delta)) - \tau^2\left(1 + \frac{\overline{s}sr(1 - \delta)}{1 - \overline{s}s - r(1 - \delta)}\right)\right)}{\left(4\beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2\right) \left(6\beta (1 - r(1 - \delta)) - \tau^2\right)^2} \\ & = & K_3 - C_{PAT}, \end{array}$$

with $K_3 > 0$.

From (8) and (10):

$$\begin{array}{lcl} V^{0,PAT} - V^{0,0} & = & (\alpha - c)^2 \Bigg((1 - r(1 - \delta)) \Bigg(\frac{\Big(6\beta (1 - r(1 - \delta)) - \tau^2 \Big)^2 - \Big(4\beta (1 - r(1 - \delta)) - \tau^2 \Big)^2}{\Big(4\beta (1 - r(1 - \delta)) - \tau^2 \Big) \Big(6\beta (1 - r(1 - \delta)) - \tau^2 \Big)^2} \Bigg) \Bigg) - C_{PAT} \\ & = & K_4 - C_{PAT}, \end{array}$$

with $K_4 > 0$.

As well as for trade marks, the benefit of filing a patent is positive if the filing costs are sufficiently low. This may not always be the case as the registration of a patent - more than a trade mark - is a relatively complex procedure and requires some human and financial resources. Therefore the costs of filing a patent may outweigh the benefits gained from the patent protection.

Considering sufficiently low levels of filing costs, we have:

$$V^{TM,PAT} > \{V^{TM,0}, V^{0,PAT}\} > V^{0,0}$$
.

The optimal IP right strategy for the innovating firm is thus always to register both a patent and a trade mark when considering negligible filing costs. Besides, the trade mark benefit differs depending on whether a patent is also filed or not. We then seek to analyse this difference in order to conclude about the complementary or susbstitue relationship between the two IP rights.

2.3.2 Complementarity analysis

In the following, we investigate in which conditions the supermodularity inequality (3) is verified, i.e. where the difference $(V^{TM,PAT} - V^{0,PAT}) - (V^{TM,0} - V^{0,0})$ is positive. This amounts to comparing the benefit of filing a trade mark in case of patent protection and in case of no patent. From (11) and (12) we deduce that:

$$V^{TM,PAT} - V^{0,PAT} - \left(V^{TM,0} - V^{0,0}\right) = (\alpha - c)^2 \left(\frac{\tau^2 r (1 - \delta) \frac{\overline{s}_S}{1 - \overline{s}_S}}{\left(4\beta \left(1 - r \frac{1 - \delta}{1 - \overline{s}_S}\right) - \tau^2\right) \left(4\beta (1 - r (1 - \delta)) - \tau^2\right)} - (1 - r (1 - \delta)) \frac{\tau^2 \overline{s} \frac{1 - s}{1 - \overline{s}_S}}{\left(6\beta (1 - r (1 - \delta)) - \tau^2\right)^2}\right).$$

We have

$$\frac{\partial \left[V^{TM,PAT} - V^{0,PAT} - \left(V^{TM,0} - V^{0,0} \right) \right]}{\partial s} = \left(\alpha - c\right)^2 \left(\frac{(1 - r(1 - \delta))\tau^2 (1 - \overline{s})\overline{s}}{\left(6\beta (1 - r(1 - \delta)) - \tau^2\right)^2 (1 - \overline{s}s)^2} + \frac{(1 - \delta)r\tau^2\overline{s}}{\left(4\beta \left(1 - r\frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2\right)^2 (1 - \overline{s}s)^2} \right).$$

This expression is always positive if $1 - r(1 - \delta) > 0$, which is always true under the conditions of existence of an equilibrium. The level of complementarity between trade marks and patents is thus increasing with the level of advertising spillovers.

For s approaching 0, we have

$$\Big(V^{TM,PAT} - V^{0,PAT}\Big) - \Big(V^{TM,0} - V^{0,0}\Big) \rightarrow (\alpha - c)^2 \left(-(1 - r(1 - \delta)) \frac{\tau^2 \frac{1 - s}{1 - \overline{s}s}}{\left(6\beta(1 - r(1 - \delta)) - \tau^2\right)^2}\right),$$

which is always negative under the conditions of existence of an equilibrium, so trade marks and patents are substitute. For s approaching 1, we have

$$\Big(V^{TM,PAT}-V^{0,PAT}\Big)-\Big(V^{TM,0}-V^{0,0}\Big) \rightarrow (\alpha-c)^2 \left(\frac{\tau^2 r(1-\delta)\frac{\overline{s}}{1-\overline{s}}}{\Big(4\beta\Big(1-r\frac{1-\delta}{1-\overline{s}}\Big)-\tau^2\Big)\Big(4\beta(1-r(1-\delta))-\tau^2\Big)}\right),$$

which is positive under the conditions of existence of an equilibrium. So for high values of s, trade marks and patents are complementary. Indeed, in the first period, trade marks provide full protection of the goodwill in case of patent protection, whereas in case of no patent protection, trade marks still allow a non null level of advertising spillovers s. The higher those advertising spillovers are, the more it is beneficial to reinforce the trade mark with a patent.

Besides, we have

$$\frac{\partial \left[V^{TM,PAT} - V^{0,PAT} - \left(V^{TM,0} - V^{0,0}\right)\right]}{\partial \delta} = (\alpha - c)^2 r \tau^2 \left(\frac{1}{\left(4\beta(1 - r(1 - \delta)) - \tau^2\right)^2} + \overline{s} \frac{1 - s}{1 - \overline{s}s} \frac{6\beta(1 - r(1 - \delta)) + \tau^2}{\left(6\beta(1 - r(1 - \delta)) - \tau^2\right)^3} - \frac{1}{1 - \overline{s}s} \frac{1}{\left(4\beta\left(1 - r\frac{1 - \delta}{1 - \overline{s}s}\right) - \tau^2\right)^2}\right).$$

This expression is negative for sufficiently high values of s, so above a certain threshold of advertising spillovers, the complementarity tends to decrease with the depreciation rate of advertising. Indeed with high depreciation rates, only a small proportion of the goodwill accumulated in the monopoly period is transferred to the second period, which tends to decrease the complementarity effect.

For δ approaching its minimum value, for which $(4\beta (1 - r (1 - \delta)) - \tau^2) = 0$, we have

$$V^{TM,PAT} - V^{0,PAT} - (V^{TM,0} - V^{0,0}) \rightarrow +\infty$$

and for δ approaching 1, we have

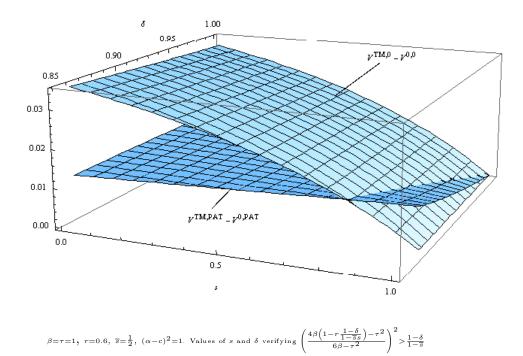
$$V^{TM,PAT} - V^{0,PAT} - \left(V^{TM,0} - V^{0,0}\right) \rightarrow -(\alpha - c)^2 \frac{\tau^2 \overline{s} \left(\frac{1-s}{1-\overline{s}s}\right)}{(6\beta - \tau^2)^2} < 0$$

So for high levels of depreciation rate of advertising, patents and trade marks tend to be substitute.

Without loss of generality, we can consider the parameters of the inverse demand function β and τ given, as well as \overline{s} , the amount of spillovers benefiting the leader. In the following, we consider $\beta = \tau = 1$, and $\overline{s} = \frac{1}{2}$. We besides attribute a definite value to the discount rate between the two periods r. A reasonable value for r is 0.6^2 . Lastly, we attribute a value of 1 to the common scaling parameter $(\alpha - c)^2$. We can then represent the level of trade mark benefits in case of patent and of no

²Assuming an annual interest rate of r_0 , and a patent period of T years, the discount rate between the two periods in the model can be approximated by $r \equiv \frac{\sum_{t=0}^T \left(\frac{1}{1+r_0}\right)^t}{\sum_{t=T+1}^\infty \left(\frac{1}{1+r_0}\right)^t} = 1 - \left(\frac{1}{1+r_0}\right)^T$. Taking $r_0 = 0.5$, and considering that the patent period lasts twenty years, we obtain a value of $r \simeq 0.6$.

Figure 1: Benefits of trade mark filing



patent according to s and δ , considering negligible costs of IP rights (Figure 1)³. The intertemporal profits resulting from each IP rights strategy are represented in Annex 1.

Under the assumptions of the model, depending on the of level of advertising spillovers s and on the level of depreciation of advertising expenditure, patents and trade marks can be found to be either complementary or substitute. The interpretation of the results is the following. The interaction between patents and trade marks is characterised by two counterbalancing effects. There is on the one hand a substitution effect. The trade mark benefits the firm only when it faces competition. As we assume that patent leads to a non-competition period, trade marks are comparatively less advantageous for the pioneer firm when there is also a patent filed. In the extreme, if the protection offered by patents was

 $^{{}^3\}text{The conditions of the model require that } \left(\frac{4\beta\left(1-r\frac{1-\delta}{1-\overline{s}s}\right)-\tau^2}{6\beta-\tau^2}\right)^2 > \frac{1-\delta}{1-\overline{s}}, \text{ so that with } \beta=\tau=1 \\ \text{and } \overline{s}=\frac{1}{2}, \text{ the model does not admit an equilibrium for } \delta<0.85.$

infinite in time, the benefit of trade mark would be null as the firm would not need to protect its brand from confusion with other firms. On the other hand, we find a complementary effect: the trade mark makes it possible to capture entirely the goodwill built during the monopoly period. The trade mark benefits in the second period will be all the more important if the firm has benefited from a monopoly period, so that trade marks in the second period are comparatively more advantageous if the firm had a patent filed in the first period.

Depending on the levels of advertising spillovers and advertising depreciation rate, either the first effect or the second effect can be predominant. For sufficiently high values of advertising spillovers and low values of advertising depreciation rate, patents and trade marks are found to be complementary. This is likely to be the case for example in pharmaceutical industries. Indeed in those sectors the depreciation of advertising tends to be low (as the products tend to have relatively long life cycles and remain stable over time). In addition in those sectors the technology is well codified, which implies that innovations can be perfectly duplicated, so that advertising performed by firms is very likely to benefit the product in general. The level of advertising spillovers in those sectors is then relatively high. In contrast, in other high-tech sectors such as computer and electronic products, the depreciation rate of advertising tends to be high (as the product life cycle is short) and advertising spillovers tend to be low (the technology being not well codified, the characteristics of the product are hardly identified by the customer, so that advertising is above all advertising for the brand). In those sectors the substitution effect tends to outweigh the complementarity effect. This does not necessarily imply that firms use only one type of IP right, as in the case of negligible registration costs the optimal strategy is always to file both a patent and a trade mark. However, in those cases the incremental benefit of using both types of IP rights instead of one is lower.

3 Empirical investigation

3.1 Tested hypotheses and methodology

This section presents some preliminary empirical evidence aiming at illustrating the theoretical model presented above. The general purpose is to test the complementarity between the use of trade marks and the use of patents by firms as tools to protect their assets. The theoretical model considers the link between patents and trade marks at the product level. However, this framework is not directly transferable to empirical analysis, since IP right data are generally not available at

the product level. Therefore, in the following, we shift the framework of analysis from product-level to firm-level. Although those frameworks are not equivalent (as firms are likely to sell several products on the market), we may assume that the IP strategies observed at the firm-level are representative of the strategy adopted by the firm at each product launch (a number of companies registering systematically new trade marks, or patents, or both each time they introduce a new product on the market). Our general empirical strategy then consists in estimating and comparing the firms' performance resulting from various IP right strategies. We use the market value of the firm as a measure of firm performance, which enables an intertemporal analysis of the effects of IP rights: assuming efficient stock markets, the firm's market value is equal to the sum of its discounted future profits, which is the target variable in our previous theoretical model. Another measure of performance, such as the present profit margin in time t, would be inadequate as the context of the model is dynamic, with inter-temporal effects of IP strategy choices.

We follow the market value approach, which combines accounting data with the valuation on the stock market. This approach has been used in particular to assess returns to innovation (Griliches 1981, Hall et al. 2000, Greenhalgh and Rogers 2007, Sandner 2009). The general idea of those models is that investors estimate a firm's value according to the returns that they expect from its assets (either tangible or intangible). The purpose of those models is to disentangle the contribution of tangible and intangible assets, intangible assets being proxied by measures of R&D, the number of patents or the number of trade marks. In our model, by contrast, the intangible assets of the firm are considered as given, and IP rights are considered in their function to appropriate the benefits of those assets. We thus seek to analyse how the IP strategy affects the profit of the firm, considering everything else equal, notably their levels of R&D and innovative activity, reflected in intangible assets. Thus we include both tangible and intangible assets in explanatory variables. We consider

$$V = qA, (13)$$

where A is the amount of firm's total assets (tangible and intangible). Taking natural logarithms on both sides of (13), the previous equation can be rewritten as $\ln(V) = \ln(q) + \ln(A)$. We assume that the coefficient q depends on the IP strategy of the firm: $q_{TM,PAT}$, $q_{TM,0}$, $q_{0,PAT}$, $q_{0,0}$.

Following the supermodularity approach (see Mohnen and Röller 2003 and Guidetti $et\ al.\ 2009$ for deeper methodological explanations on empirical tests of supermodularity), our estimation strategy is to regress the log of the market

value of the firm on the log of its assets, including the four dummies associated to the potential IP right strategies in the set of explanatory variables: use of no patent and no trade mark $(1_{0,0})$, of trade marks but no patents $(1_{TM,0})$, of patents but no trade marks $(1_{0,PAT})$, and of both patents and trade marks $(1_{TM,PAT})$. All dummies are included in the regression, which is thus "without constant". This is necessary in order to get all the estimates of coefficients and variance/covariance. The first model specification is:

$$\ln(V) = \beta_1(1_{0,0}) + \beta_2(1_{TM,0}) + \beta_3(1_{0,PAT}) + \beta_4(1_{TM,PAT}) + \gamma \ln(A)$$
(14)

Going back to the model equation $\ln(V) = \ln(q) + \ln(A)$, the coefficients β correspond to the evaluation of $\ln(q)$ corresponding to the various IP right strategies, and γ allows for non constant returns to scale. From the previous theoretical section, we derive that complementarity holds if $\beta_1 + \beta_4 > \beta_2 + \beta_3$. To investigate this, we apply a one-sided t-test with null hypothesis $H_0: \beta_1 + \beta_4 - \beta_2 - \beta_3 > 0$.

The previous specification considers the IP strategies as invariant for the firms, which are assumed to always rely on the same combination of IP rights to protect their innovations. In order to relate more precisely the returns of the firms' assets and their IP strategy, we introduce a second specification, focusing on the difference in firm's market value between two points in time (t=1 and t=2). According to the previous framework, we have:

$$\frac{V_2}{V_1} = q \frac{A_2}{A_1},\tag{15}$$

where V_t is the market value in t, and A_t is the amount of firm's total assets in t. Here the coefficient q varies depending on the IP rights acquired by the firm between the two periods t=1 and t=2. This means that the growth in market value depends on the IP strategy specifically associated to the assets acquired between the two periods. Taking the logarithms on both sides of (15), the second model specification corresponds to:

$$\ln(V_2) = \ln(V_1) + \beta_1(1_{0,0}) + \beta_2(1_{TM,0}) + \beta_3(1_{0,PAT}) + \beta_4(1_{TM,PAT}) + \gamma \ln(A_2) - \gamma \ln(A_1), \tag{16}$$

where the dummy variables correspond to the use of the corresponding IP right between t = 1 and t = 2.

The IP right strategy is likely to be dependent on the life cycle of the firm: firms tend to file more IP right applications in their early life time (protecting the name of the firm or their core technology). Thus, in the two previous specifications

we control for the age of the firm. We besides add controls for sector.

The various hypotheses tested stemming from the theoretical model are :

```
\begin{cases} H_{1a}:\beta_2>\beta_1\\ H_{1b}:\beta_3>\beta_1 &: \text{ inter-temporal profits are higher if the firm uses IP right}\\ H_{1c}:\beta_4>\beta_1 \\ \end{cases} protection.
```

 $\begin{cases} H_{1d}: \beta_4 > \beta_3 \\ H_{1e}: \beta_4 > \beta_2 \end{cases} : \text{ inter-temporal profits are higher if the firm chooses to use}$ both a patent and a trade mark than only one type of IP right.

Those hypotheses are always verified in the framework of the theoretical model if IP rights registration costs are negligible.

 $H_0: \beta_1 + \beta_4 > \beta_2 + \beta_3$: supermodularity hypothesis.

The above inequality does not depend on IP rights registration costs. According to the theoretical model prediction, the result should depend on market characteristics, and is thus likely to vary across sectors.

3.2 Data sources and descriptive statistics

Dataset building

The various tests described in the previous paragraph are performed on a firm-level database encompassing the trade marking and patenting activity of a sample of French firms listed on the stock exchange, linking several data sources. General information on firms, as well as accounting and financial variables were retrieved from the database ORBIS© (April 2011 version), edited by the Bureau Van Dijk, containing information on more than one million French business entities. Since market value is used as the dependent variable in the regression, the sample is restricted to publicly traded firms. The year considered for the estimation is 2007, before the worsening of the late 2000's financial crisis, in order to avoid the exogenous variation of stock market variables.⁴ A second reason for avoiding the crisis period is that the model focuses on firms' IP activity, and the latter is generally hampered during recession periods. Restricted to French firms for which financial and accounting data in 2007 are available, the sample contains 785 observations⁵.

⁴NB: the market value is considered at the end of 2007, a time at which the sub-prime crisis had already begun, yet with much lower impact on market prices

⁵To be consistent with the theoretical model, the sample should ideally be restricted to innovative firms. Otherwise we cannot know if firms have no IP right activity because they do not innovate (which would have a negative impact on market value compared to other firms) or because they innovate but do not protect their innovations with IP rights. One possibility to have information on innovating behaviour would be to match the dataset with innovation survey data. However, because of the small size of innovations urvey samples, this would reduce our sample size drastically (from 785 to 170 osbservations), which would not allow us to achieve sig-

The firm data were matched with data about trade marks and patents applied for at the national and European level over the period 1998-2007. National and Community trade mark applications were provided by the INPI and by the OHIM, respectively, and data on national and EPO patents were retrieved from the EPO *PATSTAT* database.

The matching methodology used consists in linking the company name in the firm database to the applicant name listed in the various IP databases, using an automatic computer-based procedure. This procedure first harmonises the names in both firm and IP datasets, to take into account possible variations in denominations that firms may use, based on the algorithm developed by Magerman, Van Looy, and Xiaoyan (2006). The matching is then done according to exact identity of the harmonized names. This matching methodology is thus quite careful, favouring the occurrence of false-negatives over false-positives in the results.⁶

Variables used and descriptive statistics

The dependent variable used in the regressions is the natural logarithm of the firm's market value, V. The market value of a firm is defined as the sum of its market capitalisation and the market value of its debt. Following Blundell *et al.* (1999), Hall and Oriani (2006), and Sandner (2009), we calculated the firm's market value as the sum of the nominal value of market capitalisation and outstanding debt. Finally, outstanding debt was calculated as the sum of long term debt and current liabilities as reported in ORBIS©.

In the set of explanatory variables we use the "total assets" as directly contained in ORBIS© database, defined as the sum of tangible and intangible assets. Although IP rights are sometimes qualified as "intangible assets", patents and trade marks applied for by the firm are not accounted in the intangible assets. The latter are recorded on balance sheets at cost, so IP rights are only included in intangible assets if they have been acquired from an external source (see International Accounting Standards Board 2007). For IP rights acquired internally, what is recorded is their corresponding investments (R&D or brand equity investments),

nificant results differentiated by sectors. Nevertheless the large majority of publicly traded firms are innovating: based on our sample of listed French firms matched with the French results of the Community Innovation Survey 2008, containing in total 170 observations, 113 (66%) have innovated in product or in service during the years 2006-2008, which is a much larger proportion than in the complete CIS sample (26%).

⁶Recently, a number of other initiatives have aimed at linking firm-level data and IP data, notably the NBER patent data project, linking USPTO patents to the Standard and Poor's Compustat database on US firms (Hall et al. 2001) and the Oxford Firm-Level Intellectual Property (OFLIP) database, linking UK firm data from the FAME database and UKIPO patents and trade marks data (Helmers, Rogers and Schautschick 2011). For an overview of other initiatives, see Helmers, Rogers and Schautschick (2011).

and not the IP right itself whose financial value is not possible to assess. This avoids the presence of an endogeneity issue in the joint inclusion of IP rights dummies and intangible assets in the set of explanatory variables in the regressions. The variable "intangible assets" in ORBIS© contains R&D, advertising and organisational expenses (see Giannetti 2003). Thus what the model captures is the respective effects of intangible investments and of the use of IP rights to protect those investments, which is in line with the theoretical framework used in Section 1.

The dummy variables corresponding to the IP strategy relate to the fact that the firm applied for at least one patent and/or at least one trade mark during the period considered. In the first specification, the period over which the IP right behaviour is tracked is 1998-2007, which we assume describes the usual IP right behaviour of the firm. In the second specification, the IP right behaviour is considered only in the years 2006-2007, since the model focuses on the difference in market value before and after this period. Table 1 gives descriptive statistics for the final dataset.

The different IP strategies are not equally represented in the sample. A large majority of firms in the sample use IP rights: 78% applied for at least one patent or one trade mark during 1998-2007, and 57% used IP rights in the only two years 2006-2007 (the proportion might be even higher since the matching methodology tends to favour false negatives). Those high shares can be explained by the fact that the sample contains only publicly listed companies, which tend to be more active in IP than the whole population of firms. The use of trade marks is much more frequent than the use of patents (76% of firms used trade marks, 33% used patents in 1998-2007). The proportion of firms using both types of IP rights in 1998-2007 is 21%, so the complementary states correspond to nearly half of the sample (43%).

3.3 Results

In this section, we estimate the market value equations based on the specifications (14) and (16) presented above.

Table 1. Desc.	riptive st	atistics ic	or the Final S	sampie	
Variable	Obs	Mean	Std. Dev.	Min	Max
Valuation and assets variables	(bil.euros)			
Market Value 2007 ¹	785	3.242	14.510	0.0005	214.77
Market Capitalisation 2007 ¹	785	1.843	9.518	0.0001	148.471
Long Term Debt 2007 ¹	785	0.505	2.147	0	32.686
Current Liabilities 2007 ¹	785	0.896	3.987	0.00005	48.692
Total Assets 2007 ¹	785	2.641	11.561	0.0003	186.149
Market Value 2005 ²	556	3.598	14.349	0.001	177.499
Market Capitalisation 2005 ²	556	1.917	8.808	0.0003	130.278
Long Term Debt 2005 ²	556	0.559	2.613	0	42.636
Current Liabilities 2005 ²	556	1.122	4.453	0	44.788
Total Assets 2005 ²	556	3.120	12.874	0.0004	171.136
\mathbf{Age}					
Age of the firm in 2007	785	38.590	42.141	0	375
IP strategy distribution	1998-2	20071	2006-2007 ²	TM 2006-2	007 / PAT 2006
TM, PAT	168 (2	21%)	72~(13%)	6	0 (11%)
TM, 0	433~(55%)		228~(41%)	240 (43%)	
0, PAT	16 (2%)		19 (3%)	14 (3%)	
0,0	168 (21%)		$237\ (43\%)$	$242\ (44\%)$	
Sector distribution ³					
High-Tech Manuf.	77 (10	$(9\%)^{1}$		4	$8 (9\%)^2$
Medium-High-Tech Manuf.	$79\ (10\%)^1$			58	$8 (10\%)^2$
Medium-Low-Tech Manuf.	ıf. 43 (5%) ¹			$39 \ (7\%)^2$	
Low-Tech Manuf.	$98 \ (12\%)^1$			78	$8 (14\%)^2$
KnowlIntensive Services	$251 (32\%)^1$			$177 (32\%)^2$	
					$(18\%)^2$

 $^{67 (9\%)^1}$ $56 \ (10\%)^2$ Other sectors

¹Sample restricted to firms for which market value in 2007 is known: 785 observations
²Sample restricted to firms for which market value in 2007 and 2005 is known: 556 observations
³Sector categories corresponding to Eurostat aggregation of manufacturing industries and services

according to R&D intensity and share of tertiary educated persons, respectively, based on NACE Rev.2.

Specification (14) is estimated by:

```
\ln V_t = \beta_1(1_{0.0}) + \beta_2(1_{TM,0}) + \beta_3(1_{0,PAT}) + \beta_4(1_{TM,PAT}) + \gamma \ln(A_t) + \sigma age + i.sector,
```

in t=2007 where V is the firm's market value, A is the amount of the firm's total assets, and i.sector corresponds to the dummy variables of the sectors (NACE Rev. 2, 2-digit level). The dummy variables of trade mark and/or patent use indicate whether the firm applied for at least one patent or one trade mark at the national or European level between 1998 and 2007 (based on the application date). We also estimated for comparison the same model with constant, omitting the dummy variable corresponding to no IP right application.

Specification (16) is estimated by:

$$\begin{split} \ln\!\left(V_{t_2}\right) &= \ln\!\left(V_{t_1}\right) + \beta_1(1_{0.0}) + \beta_2(1_{TM,0}) + \beta_3(1_{0,PAT}) + \beta_4(1_{TM,PAT}) + \gamma_1 \ln\!\left(A_{t_2}\right) - \gamma_2 \ln\!\left(A_{t_1}\right) \\ &+ \sigma age + i.sector, \end{split}$$

in $t_2 = 2007$ and $t_1 = 2005$, and where the dummy variables of trade mark and/or patent use indicate if the firm applied for at least one patent or one trade mark at the national or European level between 2006 and 2007 (application date).

Table 2 presents the results of the regressions on the complete sample. In both specifications coefficients are all significant at the 1% level, except for age, which is only significant (at 5% level) in the first specification. The global explanatory power of the model is very high, above 99% in both specifications. This is explained both by the use of without constant specification and by the very high explanatory power of the variable total asset in market value regressions (as can be seen in column 1 of the results).

The results regarding the first specification (presented in column 4) tend to be in line with the theoretical model predictions. The order of the coefficients for IP right variables are consistent with the expectations: the one-sided t-tests give significant positive results, except for $1_{0,PAT}>1_{0,0}$ and $1_{TM,PAT}>1_{0,PAT}$, for which the results are not significant. This is also supported by the results of the regression without constant (column 3), where all IP dummies have positive coefficients, significant at 1% level except $1_{0,PAT}$. In the second specification (column 6), the tests also tend to give positive results, although they are generally not significant (significant at 10% level only for $1_{TM,PAT} > 1_{0,0}$). We also estimated the second specification considering only patent applications in 2006, in order to take into account a possible delay between patent and trademark applications. The results are stable whether

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable : ln (M	arket Value 2007)				
I (TI + I A + 2007)	0.915**		0.914**	0.914**	0.937**	0.879*>
ln (Total Assets 2007)	(0.010)		(0.010)	(0.010)	(0.014)	(0.031)
1 (00.07)				-5.793**	-6.111**	
$1_{0,0} \ (98-07)$				(0.162)	(0.196)	
$1_{0.PAT} (98-07)$		0.061	0.079	-5.715**	-6.192**	
10,PAT (90-01)		(0.449)	(0.150)	(0.220)	(0.199)	
$1_{TM,0} (98-07)$		0.673**	0.184**	-5.609**	-5.898**	
$1_{TM,0} (96-07)$		(0.206)	(0.045)	(0.160)	(0.188)	
1 (08.07)		1.488**	0.254**	-5.540**	-5.838**	
$1_{TM,PAT} (98-07)$		(0.263)	(0.051)	(0.164)	(0.195)	
одо			-0.001*	-0.001*	-0.001	0.000
age			(0.000)	(0.000)	(0.001)	(0.000
$\ln(\mathrm{MV2005})$						0.735*
III (M V 2005)						(0.042
In (Total Assets 2005)						-0.631*
iii (Totai Assets 2005)						(0.047
1 _{0,0} (06-07)						-1.409*
10,0 (00-07)						(0.293
$1_{0,PAT} (06-07)$						-1.407*
10,PAT (00-01)						(0.296
$1_{TM,0} (06-07)$						-1.380*
$1_{TM,0} (00-07)$						(0.289)
1 (00.07)						-1.334*
$1_{TM,PAT} (06-07)$						(0.284
	-5.712**	4.646**	-5.793**			
constant	(0.138)	(0.933)	(0.162)			
N	785	785	785	785	556	556
R-sq	0.960	0.220	0.962	0.994	0.995	0.998
$1_{TM,0} > 1_{0,0}$				4.10**		0.92
$1_{0,PAT} > 1_{0,0}$				0.53		0.03
$1_{0,PAT} > 1_{0,0}$ $1_{TM,PAT} > 1_{0,0}$				4.96**		1.64+
$1_{TM,PAT} > 1_{0,0}$ $1_{TM,PAT} > 1_{0,PAT}$				1.22		1.04
$1_{TM,PAT} > 1_{0,PAT}$ $1_{TM,PAT} > 1_{TM,0}$				1.64+		1.04
Complementarity test: one-	sided Student te	st (t statistics): $H_0: 1_{TM,P}$	$a_{AT} - 1_{0.PAT}$	$> 1_{TM,0} - 1_{0}$	0
r		(, 0 -1 111,1-	-0.06	. 1111,0 -0,	0.58

 $\overline{OLS\ robust\ estimates.\ Standard\ errors\ in\ parentheses} \\ **\ p < 0.01,\ **\ p < 0.05,\ +\ p < 0.1$

 $All\ regressions\ also\ contain\ controls\ for\ sector\ at\ the\ Nace\ Rev. 2\ 2-digit\ level$

considering patent applications in the two years or only in 2006 (see results in Annex 2).

To investigate if the complementarity hypothesis holds, we apply a one-sided t-test on the obtained coefficients, with null hypothesis: $H_0: \beta_1+\beta_4-\beta_2-\beta_3>0$. The one-sided t-test rejects the null hypothesis at 5% level if the value of the t statistic is lower than -1.645 (then substitutability (non strict) holds). If the value of the t statistic is higher than 1.645, then strict complementarity holds at 5% level (the previous threshold is 1.282 at 10% level, and 2.326 at 1% level). The complementarity test does not give any significant result on the total sample. This could be expected since the theoretical model indicates that the results are likely to vary across sectors. To investigate this hypothesis, we estimated the previous model on sub-samples corresponding to two different sectors, both highly innovative: pharmaceutical and chemical products on the one hand and high-tech business sectors (manufacture of computer, electronic and optical products and of electrical equipment) on the other hand. The results are presented in Table 3^7 .

We find that the results of the supermodularity test vary across sectors. In pharmaceutical and chemical sectors, the test tends to be in favour of the complementarity hypothesis (at 1% level in the second specification). We find that in those sectors the benefit of filing a trade mark alone is not significant, whereas the effect of filing a trade mark and a patent is significantly higher than the effect of filing a patent alone. Similarly, filing a patent alone tends to have a negative impact on market value, whereas filing a patent jointly with a trade mark tends to have a higher impact on performance than filing a trade mark alone. In pharmaceutical and chemical sectors indeed, innovation often consists in launching new drugs or chemical products based on new molecules, and competitors are generally able to launch perfect substitutes on the market. In this situation, advertising is for a large part likely to be advertising for the product in general, so that it is not easily appropriable by the firm even if the latter registers a trade mark. Besides, drugs and chemical products tend to have relatively long life cycles, so that the advertising depreciation rate over time is likely to be relatively low. In those types of sectors, the theoretical model predicts that the complementarity effect tends to outweigh the substitution effect so that it is in the firms' interest to use patents jointly with trade marks in order to build goodwill during the monopoly period

⁷Pharma and Chemicals correspond to firms in NACE Rev. 2 sectors 20 (manuf. of chemicals and chemical products), 21 (manuf. of basic pharmaceutical products and pharmaceutical preparations), and 86 (human health activities). Computer and electrical equipment correspond to firms in NACE Rev. 2 sectors 26 (manuf. of computer, electronic and optical products) and 27 (manuf. of electrical equipment).

Table 3: Market value regression and one-sided t-tests on pharma/chemicals and and high-tech business sectors

	(1)	(a)	(4)	(5)
	(1) Pharma &	(2) Pharma &	Computer	Computer
	Chemicals	Chemicals	& elec.	& elec.
			equipment	equipment
Dependent variable : ln (Market Value 200	07)		
n (Total Assets 2007)		0.862**	0.886**	0.829**
ii (Total Assets 2001)		(0.212)	(0.033)	(0.072)
1 _{0,0} (98-07)	-5.678**		-5.410**	
10,0 (96-07)	(0.591)		(0.386)	
1 (08 07)	-5.654**		-5.255**	
$1_{0,PAT} (98-07)$	(0.433)		(0.355)	
$1_{TM,0} (98-07)$	-5.225**		-5.189**	
1TM,0 (90-01)	(0.519)		(0.380)	
$1_{TM,PAT} (98-07)$	-5.001**		-5.130**	
1TM,PAT (90-01)	(0.475)		(0.356)	
	-0.003 +	0.001	0.001	0.005*
age	(0.002)	(0.002)	(0.002)	(0.002)
- (MN/200E)		1.055**		0.811**
n(MV2005)		(0.211)		(0.134)
n (Total Assets 2005)		-0.945**		-0.710**
n (Total Assets 2005)		(0.183)		(0.156)
1 (00.07)		0.742		-0.594
1 _{0,0} (06-07)		(1.726)		(0.789)
1 (00.07)		-0.029		-0.156
$1_{0,PAT} \ (06-07)$		(1.444)		(0.792)
1 (00.07)		0.615		-0.717
$1_{TM,0} \ (06-07)$		(1.590)		(0.773)
1 (00.07)		0.816		-0.564
$1_{TM,PAT} (06-07)$		(1.589)		(0.755)
N	49	31	72	47
R-sq	0.993	0.998	0.991	0.998
One-sided Student test: t	statistic			
$1_{TM,0} > 1_{0,0}$	1.22	-0.58	1.01	-1.24
$1_{0,PAT} > 1_{0,0}$	0.07	-2.46**	0.64	3.45**
$1_{TM,PAT} > 1_{0,0}$	1.92*	0.30	1.30 +	0.33
$1_{TM,PAT} > 1_{0,PAT}$	6.00	3.60**	0.77	-2.83**
$1_{TM,PAT} > 1_{TM,0}$	1.53 +	1.55 +	0.49	1.36 +
Complementarity test: H_0	$:1_{TM,PAT}-1_{0,1}$	$PAT > 1_{TM,0}$	- 10,0	
	0.51	2.83**	-0.36	-1.86*
		Complem.		Substitut.
	=	(0.01	-	(0.05
		level)		level)

 $OLS\ robust\ estimates.\ Standard\ errors\ in\ parentheses$

^{**} p < 0.01, * p < 0.05, + p < 0.1

and continue to benefit from it after the expiration of the patent.

In computer and electrical equipment sectors, by contrast, the supermodularity test in the second specification tends to be in favour of substitutability. In those sectors we find that filing a patent alone has a significant positive impact on market value, whereas filing trade marks has no positive impact. Filing a patent jointly with a trade mark besides tends to have a lower impact on market value than filing a patent alone. This suggests that in those sectors the crucial asset to be protected is the technology, and that it is on the contrary not beneficial to invest in the protection of goodwill. This might be explained by the fact that in those sectors, relying on cutting-edge technology, the depreciation rate of products and therefore of advertising tends to be very high. The patent period is then likely to cover the major part of the life cycle of the technology, so that products are less likely to be imitated and trade mark protection is less needed. In that case, the substitution effect tends to outweigh the complementarity effect.

4 Conclusion

In the paper by Amara et al. (2008), which shows complementarities between the use of various IP protection mechanisms for firms in KIBS sectors the authors call for future research on the factors that could explain those complementarities. One of those factors is the interaction of the legal mechanisms themselves. The main contribution of the present paper is to assess the interrelated effects of IP rights considering them in their core function as legal protection devices instead of as proxies of other underlying assets. We tackle this question both through a formal theoretical model and through an empirical analysis. Using a basic modelling approach, we compare the outcome of adopting various IP right strategies for innovating firms that commercialise their own innovation: patent or not and/or trade mark or not, and then assess the complementarity or substitutability relationship between the two IP rights based on the supermodularity approach.

The main finding of our model is that the complementary or substitute relationship between trade marks and patents is not straightforward. We find that the interaction between the two IP rights is characterized by two counterbalancing effects: a temporal substitution effect – as the patent period reduces the time during which the firm faces competition and needs a trade mark to protect its reputation against other firms - and a complementarity effect – as the trade mark enables the firm to extend the reputational benefits of the monopoly period beyond the

expiration of the patent. We show that the predominance of one or the other effect depends on exogenous parameters, especially the levels of advertising depreciation rate and spillovers. If the spillovers are low and the depreciation rate is high, for example in sectors such as high-tech business sectors, then trade marks are likely to be substitutes, so the benefits of registering a trade mark will be all the more important if the firm cannot register a patent. In contrast, if advertising spillovers are high and the advertising depreciation rate is low, for example in sectors such as pharmaceutical or chemical products, then trade marks and patents are likely to be complementary. The optimal IP right strategy of firms may then vary from one context to another, from one firm to another. Following the conclusion of Teece (1986) that the profit gained from innovation depends on the possibility of the firm to use complementary assets, our model goes a step further and states that the relationship between the various assets is itself dependent on the context in which the firms operate.

The implications of this model are twofold. First, there are implications for IP right management within firms. We show that beyond the question of the eligibility of the innovation to the various types of IP rights, the profitability of a diversified IP strategy depends on context elements, which need to be taken into account to determine the benefits and costs of the various combinations. Failure to identify complementarity (resp. substitutability) between some protection mechanisms may lead to underexploitation (resp. overexploitation) of synergies and underprotection (resp. overprotection) of innovations. Secondly the model has implications for economic analyses. Whenever investigating firms' IP activity, for example as a proxy for other intangible assets, one should bear in mind the existence of context-dependent interaction effects between the various types of protection.

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Annexes

Annex 1: Outcomes of the various IP strategies according to s and δ

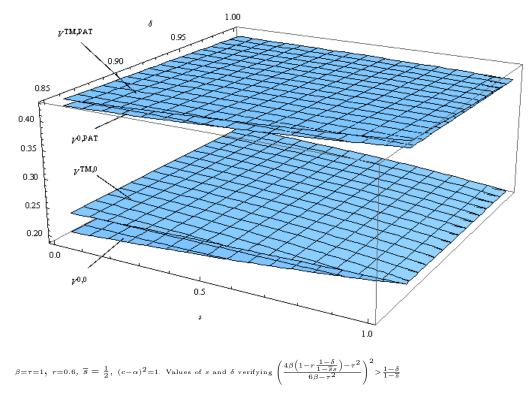


Figure 2: Intertemporal profits resulting from the various IP strategies

Annex 2: Estimation of the second specification considering 2006 patent applications

Table 4.	Moulrot	110 1110	regression	and	one sided	t toata
Table 4	Viarket	value	regression	and	one-sided	T-Tests

Variables	All sample	Pharma & Chemicals	Computer & electrical equipment
Dependent variable : ln (M	arket Value 2007	.)	equipment
ln (Total Assets 2007)			
	0.879**	0.904**	0.827**
	(0.031)	(0.202)	(0.071)
age	0.000	0.001	0.005**
	(0.000)	(0.002)	(0.002)
$\ln(\text{MV}2005)$			
	0.736**	1.063**	0.793**
	(0.042)	(0.213)	(0.150)
ln (Total Assets 2005)	-0.631**	-0.999**	-0.688**
	(0.047)	(0.158)	(0.169)
1 _{0,0} (06-07,06)	(,	(/	()
, , , , ,	-1.415**	0.847	-0.736
	(0.295)	(1.749)	(0.893)
$1_{0,PAT} (06-07,06)$	-1.421**	0.077	-0.301
	(0.304)	(1.457)	(0.895)
$1_{TM,0}$ (06-07,06)	(0.004)	(1.101)	(0.050)
-1 1/1,0 (00 01,00)	-1.382**	0.701	-0.791
	(0.290)	(1.608)	(0.862)
$1_{TM,PAT}$ (06-07,06)	-1.356**	0.931	-0.795
	(0.287)	(1.609)	(0.864)
N	(0.201)	(1.003)	(0.004)
	556	31	47
R-sq			
0 1110 1	0.998	0.998	0.997
One-sided Student test: t s	tatistic		
$1_{TM,0} > 1_{0,0}$	1.07	-0.66	-0.54
$1_{0,PAT} > 1_{0,0}$	-0.08	-2.41**	3.50**
$1_{TM,PAT} > 1_{0,0}$	1.44 +	0.34	-0.89
$1_{TM,PAT} > 1_{0,PAT}$	0.78	3.75**	-4.05**
$1_{TM,PAT} > 1_{TM,0}$	0.61	1.88*	-0.05
Complementarity test: H_0	$: 1_{TM,PAT} - 1_0,$	$_{PAT} > 1_{TM,0}$	$-1_{0,0}$
	0.36	2.94**	-3.62**
	36	=:= *	Substitut.
	=	Complem.	(0.01
		(0.01	level)
		level)	*

OLS robust estimates. Standard errors in parentheses ** p < 0.01, * p < 0.05, + p < 0.1

 $The\ regression\ on\ the\ whole\ sample\ also\ contain\ controls\ for\ sector\ at\ the\ Nace\ Rev. 2\ 2-digit$ level